**Contents**

* Introduction
* Literature Review
* Objective
* Mathematical Formulation
* Reference

**Introduction**

Over the past few decades, because of the presence of various dissipative behaviors such as flow separation and reattachment, multiple regions of fluid flow recirculation, etc. flow through abrupt change in flow geometry has put forward the researchers for extensive research. Suddenly expanded and contracted geometry have immense importance to researchers involved in bioengineering and bio-medical area. These types of phenomena may occur in veins or arterial system due to some diseases. One of the most important diseases of veins or arterial system is stenosis, which is a very interesting and demanding research area in biomedical engineering at present.

A stenosis is an abnormal narrowing in a [blood vessel](https://en.wikipedia.org/wiki/Blood_vessel) or other tubular [organ](https://en.wikipedia.org/wiki/Organ_(anatomy)) or structure.

* **Types of Stenosis:**

1. According to the  vascular stenotic lesions, there are peripheral, coronary, carotid, renal artery stenosis.

* **Peripheral artery stenosis:** These types of stenosis occurs during exercise and systems are muscle pain.
* **Coronary artery stenosis:** These is the sensation of [chest pain](https://en.wikipedia.org/wiki/Chest_pain), pressure, or squeezing, often due to [not enough blood flow](https://en.wikipedia.org/wiki/Ischemia) to the [heart muscle](https://en.wikipedia.org/wiki/Cardiac_muscle) as a result of [obstruction](https://en.wikipedia.org/wiki/Vascular_occlusion) or [spasm](https://en.wikipedia.org/wiki/Vasospasm) of the coronary arteries.
* [**Carotid artery stenosis**](https://en.wikipedia.org/wiki/Carotid_artery_stenosis)**:** Carotid stenosis is a narrowing or constriction of the inner surface ([lumen](https://en.wikipedia.org/wiki/Lumen_(anatomy))) of the [carotid artery](https://en.wikipedia.org/wiki/Carotid_artery), usually caused by arteriosclerotic vascular disease.
* [**Renal artery stenosis**](https://en.wikipedia.org/wiki/Renal_artery_stenosis)**:** Renal artery stenosis is the [narrowing](https://en.wikipedia.org/wiki/Stenosis) of one of the renal arteries. This narrowing of the renal artery can impede [blood flow](https://en.wikipedia.org/wiki/Blood_flow) to the target [kidney](https://en.wikipedia.org/wiki/Kidney), resulting of a [high blood pressure](https://en.wikipedia.org/wiki/Hypertension)

1. According to the  stenoses in [heart valves](https://en.wikipedia.org/wiki/Heart_valve) are pulmonary, mitral, tricuspid, aortic valve stenosis.

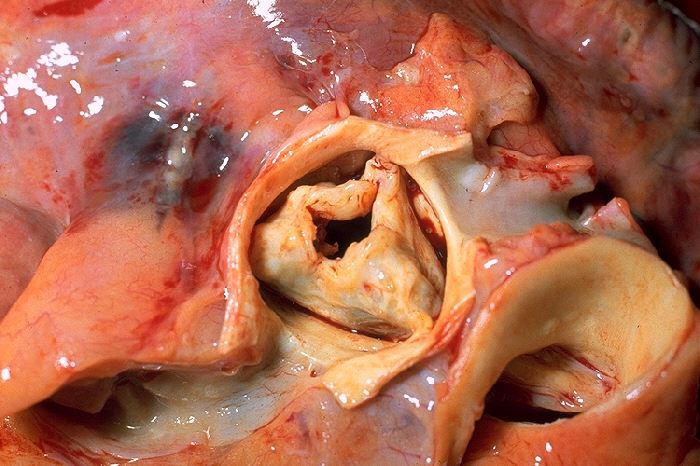
* [**Pulmonary valve stenosis**](https://en.wikipedia.org/wiki/Pulmonary_valve_stenosis)**:** These are the thickening of the [pulmonary valve](https://en.wikipedia.org/wiki/Pulmonary_valve).
* [**Mitral valve stenosis**](https://en.wikipedia.org/wiki/Mitral_valve_stenosis)**:** These are the thickening of the [mitral valve](https://en.wikipedia.org/wiki/Mitral_valve) (of the [left heart](https://en.wikipedia.org/wiki/Left_heart)).
* [**Tricuspid valve stenosis**](https://en.wikipedia.org/wiki/Tricuspid_valve_stenosis)**:** These are the thickening of the [tricuspid valve](https://en.wikipedia.org/wiki/Tricuspid_valve) (of the [right heart](https://en.wikipedia.org/wiki/Right_heart)).
* [**Aortic valve stenosis**](https://en.wikipedia.org/wiki/Aortic_valve_stenosis)**:** These are the thickening of the [aortic valve](https://en.wikipedia.org/wiki/Aortic_valve).

1. There are other stenosis according to their organ position.

* [**Pyloric stenosis**](https://en.wikipedia.org/wiki/Pyloric_stenosis)
* [**Lumbar**](https://en.wikipedia.org/wiki/Lumbar_spinal_stenosis) **stenosis**
* [**Cervical**](https://en.wikipedia.org/wiki/Cervical_spinal_stenosis) **stenosis**
* [**Spinal stenosis**](https://en.wikipedia.org/wiki/Spinal_stenosis)
* [**Subglottic stenosis**](https://en.wikipedia.org/wiki/Subglottic_stenosis)
* [**Tracheal stenosis**](https://en.wikipedia.org/wiki/Tracheal_stenosis)

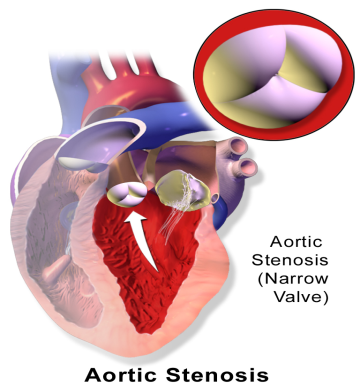
since we have focused our study in details on aortic stenosis only.

**AORTIC STENOSIS (AoS):**



Aortic stenosis (AS or AoS) is the [narrowing](https://en.wikipedia.org/wiki/Stenosis) of the exit of the [left ventricle](https://en.wikipedia.org/wiki/Left_ventricle) of the [heart](https://en.wikipedia.org/wiki/Heart) (where the [aorta](https://en.wikipedia.org/wiki/Aorta) begins), such that problems result. It may occur at the [aortic valve](https://en.wikipedia.org/wiki/Aortic_valve) as well as above and below this level. It typically gets worse over time

* **Causes of Aortic Stenosis:**

Aortic stenosis is most commonly caused by age-related progressive calcification (>50% of cases) with a mean age of 65 to 70 years. Another major cause of aortic stenosis is the calcification of a congenital [bicuspid aortic valve](https://en.wikipedia.org/wiki/Bicuspid_aortic_valve) (30-40% of cases) typically presenting earlier, in those aged 40+ to 50+.

Rare causes of aortic stenosis include [Fabry disease](https://en.wikipedia.org/wiki/Fabry_disease" \o "Fabry disease), [systemic lupus erythematosus](https://en.wikipedia.org/wiki/Systemic_lupus_erythematosus), [Paget disease](https://en.wikipedia.org/wiki/Paget_disease_of_bone), [high blood uric acid levels](https://en.wikipedia.org/wiki/Hyperuricemia), and [infection](https://en.wikipedia.org/wiki/Infection).

* **Symptoms of Aortic Stenosis:**

[Symptoms](https://en.wikipedia.org/wiki/Symptom) related to aortic stenosis depend on the degree of [stenosis](https://en.wikipedia.org/wiki/Stenosis" \o "Stenosis). Most people with mild to moderate aortic stenosis do not have symptoms. Symptoms usually present in individuals with severe aortic stenosis, though they may occur in those with mild to moderate aortic stenosis as well.

1. **Angina:**

Angina in the setting of AS occurs due to [left ventricular hypertrophy](https://en.wikipedia.org/wiki/Left_ventricular_hypertrophy) (LVH) that is caused by the constant production of increased pressure required to overcome the pressure gradient caused by the AS. While the [muscular layer](https://en.wikipedia.org/wiki/Myocardium) of the left ventricle thickens, the arteries that supply the muscle do not get significantly longer or bigger, so the muscle may not receive enough blood supply to meet its oxygen requirement.

1. **Syncope:**

It is unclear why aortic stenosis causes syncope. One popular theory is that severe AS produces a nearly fixed [cardiac output](https://en.wikipedia.org/wiki/Cardiac_output).[[10]](https://en.wikipedia.org/wiki/Aortic_stenosis#cite_note-10) When a person with aortic stenosis exercises, their [peripheral vascular resistance](https://en.wikipedia.org/wiki/Peripheral_vascular_resistance) will decrease as the blood vessels of the [skeletal muscles](https://en.wikipedia.org/wiki/Skeletal_muscles) dilate to allow the muscles to receive more blood to allow them to do more work. This decrease in peripheral vascular resistance is normally compensated for by an increase in the cardiac output. Since people with severe AS cannot increase their cardiac output, the blood pressure falls and the person will faint due to decreased blood perfusion to the [brain](https://en.wikipedia.org/wiki/Brain).

### Congestive heart failure:

[Congestive heart failure](https://en.wikipedia.org/wiki/Congestive_heart_failure) (CHF) carries a grave prognosis in people with AS. People with CHF attributable to AS have a 2-year mortality rate of 50% if the aortic valve is not replaced. CHF in the setting of AS is due to a combination of [left ventricular hypertrophy](https://en.wikipedia.org/wiki/Left_ventricular_hypertrophy) with fibrosis, systolic dysfunction (a decrease in the [ejection fraction](https://en.wikipedia.org/wiki/Ejection_fraction)) and [diastolic dysfunction](https://en.wikipedia.org/wiki/Diastolic_dysfunction) (elevated filling pressure of the LV).

### Associated symptoms:

In [Heyde's syndrome](https://en.wikipedia.org/wiki/Heyde%27s_syndrome" \o "Heyde's syndrome), aortic stenosis is associated with [gastrointestinal bleeding](https://en.wikipedia.org/wiki/Gastrointestinal_bleeding) due to [angiodysplasia](https://en.wikipedia.org/wiki/Angiodysplasia" \o "Angiodysplasia) of the [colon](https://en.wikipedia.org/wiki/Colon_(anatomy)). Recent research has shown that the stenosis causes a form of [von Willebrand disease](https://en.wikipedia.org/wiki/Von_Willebrand_disease) by breaking down its associated [coagulation](https://en.wikipedia.org/wiki/Coagulation) factor ([factor VIII](https://en.wikipedia.org/wiki/Factor_VIII)-associated antigen, also called [von Willebrand factor](https://en.wikipedia.org/wiki/Von_Willebrand_factor)), due to increased turbulence around the stenotic valve.

* **Treatment of Aortic Stenosis:**

Treatment is generally not necessary in people without symptoms.[ In [moderate](https://en.wikipedia.org/wiki/Aortic_stenosis#Echocardiogram) cases, echocardiography is performed every 1–2 years to monitor the progression, possibly complemented with a [cardiac stress test](https://en.wikipedia.org/wiki/Cardiac_stress_test). In severe cases, echocardiography is performed every 3–6 months. In both moderate and mild cases, the person should immediately make a revisit or be admitted for [inpatient](https://en.wikipedia.org/wiki/Inpatient) care if any new related symptoms appear.

1. **Medication:**

The effect of [statins](https://en.wikipedia.org/wiki/Statins" \o "Statins) on the progression of AS is unclear. The latest trials do not show any benefit in slowing AS progression, but did demonstrate a decrease in ischemic cardiovascular events.

In general, medical therapy has relatively poor efficacy in treating aortic stenosis. However, it may be useful to manage commonly coexisting conditions that correlate with aortic stenosis:

* Any angina is generally treated with [beta-blockers](https://en.wikipedia.org/wiki/Beta-blocker) and/or [calcium blockers](https://en.wikipedia.org/wiki/Calcium_blocker). [Nitrates](https://en.wikipedia.org/wiki/Nitrates) are contraindicated due to their potential to cause profound [hypotension](https://en.wikipedia.org/wiki/Hypotension) in aortic stenosis.
* Any [hypertension](https://en.wikipedia.org/wiki/Hypertension) is treated aggressively, but caution must be taken in administering [beta-blockers](https://en.wikipedia.org/wiki/Beta-blocker).
* Any [heart failure](https://en.wikipedia.org/wiki/Heart_failure) is generally treated with [digoxin](https://en.wikipedia.org/wiki/Digoxin" \o "Digoxin) and [diuretics](https://en.wikipedia.org/wiki/Diuretic), and, if not contraindicated, cautious administration of [ACE inhibitors](https://en.wikipedia.org/wiki/ACE_inhibitor).

While [observational studies](https://en.wikipedia.org/wiki/Observational_studies) demonstrated an association between lowered cholesterol with [statins](https://en.wikipedia.org/wiki/Statin" \o "Statin) and decreased progression, a [randomized clinical trial](https://en.wikipedia.org/wiki/Randomized_clinical_trial) published in 2005 failed to find any effect on calcific aortic stenosis. A 2007 study did demonstrate a slowing of aortic stenosis with the statin [rosuvastatin](https://en.wikipedia.org/wiki/Rosuvastatin" \o "Rosuvastatin).

1. **Aortic valve repair:**

Aortic valve repair or aortic valve reconstruction describes the reconstruction of both form and function of the native and dysfunctioning aortic valve. Most frequently it is applied for the treatment of aortic regurgitation. It can also become necessary for the treatment of an aortic aneurysm, less frequently for congenital aortic stenosis.

1. **Aortic valve replacement:**

In adults, symptomatic severe aortic stenosis usually requires [aortic valve replacement](https://en.wikipedia.org/wiki/Aortic_valve_replacement) (AVR). While AVR has been the standard of care for aortic stenosis for several decades, currently aortic valve replacement approaches include open heart surgery, minimally invasive cardiac surgery (MICS) and minimally invasive catheter-based (percutaneous) aortic valve replacement. However, surgical aortic valve replacement is well studied and generally has a good and well established longer term prognosis.

A diseased aortic valve is most commonly replaced using a surgical procedure with either a mechanical or a tissue valve. The procedure is done either in an open-heart surgical procedure or, in a smaller but growing number of cases, a minimally invasive cardiac surgery (MICS) procedure.

1. **Transcatheter aortic valve replacement:**

Globally more than 40,000 people have received [transcatheter aortic valve replacement](https://en.wikipedia.org/wiki/Percutaneous_aortic_valve_replacement" \o "Percutaneous aortic valve replacement) (TAVR). For people who are not candidates for surgical valve replacement and most patients who are older than 75, TAVR may be a suitable alternative.

1. **Balloon valvuloplasty:**

For infants and children, [balloon valvuloplasty](https://en.wikipedia.org/wiki/Balloon_valvuloplasty), where a balloon is inflated to stretch the valve and allow greater flow, may also be effective. In adults, however, it is generally ineffective, as the valve tends to return to a stenosed state. The surgeon will make a small incision at the top of the people's leg and proceed to insert the balloon into the artery. The balloon is then advanced up to the valve and is inflated to stretch the valve open.

1. **Heart failure:**

[Acute decompensated heart failure](https://en.wikipedia.org/wiki/Acute_decompensated_heart_failure) due to AS may be temporarily managed by an [intra-aortic balloon pump](https://en.wikipedia.org/wiki/Intra-aortic_balloon_pump) while pending surgery. In those with high blood pressure [nitroprusside](https://en.wikipedia.org/wiki/Nitroprusside" \o "Nitroprusside) may be carefully used. [Phenylephrine](https://en.wikipedia.org/wiki/Phenylephrine" \o "Phenylephrine) may be used in those with very low blood pressure.

**Literature Review**

An attempt has been taken to carry out the literatures on the thesis topic i.e. ‘a numerical study on flow through stenosis artery’ in this section. A survey of available literatures is carried out and the following is the outcome of the said activities.

**Bluestein *et al*. [1996]** have done both experimental and numerical methods to study steady in-vitro flow patterns through a model aneurysm under laminar and turbulent flow conditions over Reynolds numbers ranging from 300 to 3600. They have correlated the numerical results with experimental one which have been obtained under similar flow conditions. They have revealed that the fluid dynamics characterizing the recirculation zone formed inside the aneurysm cavity create conditions in promoting thrombus formation and the viability of rupture. They have also noted that wall shear stress values in the recirculation zone are around one order of magnitude less than in the entrance zone and a pronounced wall shear stress peak at the distal end of the aneurysm. They have also reported that under turbulent flow condition, the wall shear stress values are larger than those for the laminar cases.

**BourhanTashtoush & Ahmad Magableh *et al.* [2007]** have introduced a mathematical model of the multi stenosis inside the arteries with considering heat and ﬂuid ﬂow characteristics of blood ﬂow in multi-stenosis arteries in the presence of magnetic ﬁeld. They have solved the governing equations in terms of vorticity-stream function along with their boundary conditions with the help of a ﬁnite difference scheme and also investigated the effect of magnetic ﬁeld and the degree of stenosis on wall shear stress and Nusselt number . They have found that magnetic ﬁeld modiﬁes the ﬂow patterns and increases the heat transfer rat and the severity of the stenosis affects the wall shear stress characteristics signiﬁcantly. At last they have concluded that the magnetic ﬁeld torque will increase the thermal boundary layer thickness and the temperature gradient in the streaming blood, and hence increasing the local Nusselt number.

**Budwig *et al*. [1993]** have performed both experimental and numerical simulation to determine: (i) the overall features of the flow, (ii) the stresses on the aneurysm walls in laminar flow, and (iii) the onset and characteristics of turbulent flow. They have employed a laser Doppler Velocimetry (LDV) measurements for experimental and a commercially available numerical code FIDAP for numerical analysis. They have done a steady flow analysis for four aneurysm sizes over Reynolds numbers from 500 to 2600. They have characterized the laminar flow field by a jet of fluid (passing directly through the aneurysm) surrounded by a recirculating vortex. They have seen that the wall shear stress magnitude in the recirculation zone is about less than in the entrance zone. They have also noted that the wall shear stress and wall normal stress profiles exhibit large magnitude peaks near the reattachment point at the distal end of the aneurysm. They have also noted that the recirculation zone breaks down when there is turbulence in the aneurysm and the wall shear stress return to magnitude comparable to the entrance zone.

**Callanan *et al*. [2012]** have done finite element and photoelastic modelling to determine the accuracy of FEA in AAA strain prediction and subsequent stress analysis. They have compared experimental photoelastic method and finite element techniques using an idealised AAA geometry. The good agreement between the FEA and experimental results provides improved confidence in using FEA as a predictor for strain distribution in AAA models. They have seen that maximum strains do not occur at the location of maximum diameter. They have also seen that the roles played by strains in patient AAAs due to the curvature effect using FEA methods could be used as a clinical tool and aid the clinician in the timing of surgical intervention.

**Daniel N. Riahi *et al.* [2015]** has considered the problem of unsteady two-phase blood flow in a catheterized elastic artery with stenosis, where the form and extent of the stenosis has been chosen based on the available experimental data for a human’s artery with stenosis. He has solved equations for the dependent variables of the two-phase arterial blood flow for the elastic wall displacements subjected to reasonable modelling and approximation. He has find, in particular, that the radial and axial displacements of the elastic wall oscillate in time, and their oscillations grow with increasing the hematocrit, catheter size, volume flow rate, blood pressure force and the viscoelastic wall stress, but such oscillations decay with increasing the elastic wall stiffness and the elastic wall thickness.

**Ellahi *et al.* [2014]** have performed a theoretical analysis of blood flow of nanofluid through composite stenosed arteries with permeable walls. They have simplified the highly nonlinear momentum equations of nanofluid model by considering the mild stenosis case. From the velocity profile they havecalculated the exact solution and expressed flow impedance pressure, gradient, stream function. While performing the analysis they have taken the ratio between the radius of normal artery to the length of the artery stenosis is 1 for the case of mild stenosis. They have plotted variation in impedence against Darcy number for different Brownian motion parameter. Finally they have concluded that stress is directly proportional to the stenosis height.

**Finol and Amon *et al.* [2002]** have considered a two-aneurysm, axisymmetric, rigid wall AAA model in a numerical work to study steady flow field for the range of Reynolds numbers from 10 to 2265. They have investigated the effect of blood flow patterns and hemodynamic stresses on the model, using the spectral element method. And they have established a correlation between maximum values of hemodynamic stresses and Reynolds number. They have concluded that the spatial distribution of Wall Shear Stress Gradient (WSSG) may cause damage to the arterial wall at an intermediate stage of AAA growth.

**Gaillard and Deplano *et al.* [2005]** have performed an experimental analysis to show the effects of different flow conditions (rest and exercise), on hemodynamics in AAA. They have employed Particule Image Velocimetry (PIV) in the experimental simulation. They have noted that two vortex pairs occur at the beginning of the systolic deceleration in the proximal part of the AAA, one near the anterior wall and one near the posterior wall, for the three flow conditions. The velocity at the separation point is two times higher at moderate exercise and three times higher at intensive exercise than at rest. They have concluded that, at exercise, two areas seem to be favourable to rupture: one in the end of the distal part of the aneurysm, near the anterior wall, and one near the posterior wall.

**Gataulin*et et al*. [2015]** have performed the investigation of weakly swirling flow in a model of a blood vessel with asymmetrical stenosis using both experimental flow measurement techniques (ultrasound Doppler) and computational fluid dynamics methods. They have shown their special attention to getting data for the length of the reverse-flow zone occurring past the stenosis. They have established that the laminar steady-state flow model should be acceptable for numerical analysis of flow past the given-geometry stenosis at Reynolds number values less than 300.They have prefered at higher values of this parameter, application of the semi-empirical k-ω SST turbulence model. They have shown that flow swirl can lead to an increase of the reverse-flow zone.

**Girija Bai and Naidu *et al.* [2013]** have performed a numerical simulation of hemodynamics in blood vessels with multiple fusiform aneurysms. They investigate hemodynamic factors such as velocity and pressure. The problem has been solved by finite volume method using the CFD softwares Fluent and Gambit. They have seen that the pressure magnitude is high in the region of aneurysm, which is reverse to velocity profile

**Jun-wei *et al.* [2008]** have performed a numerical simulation on the hemodynamics of an elastic aneurysm and show the differences of the simulation result between elastic and rigid wall model. They have numerically simulated of elastic aneurysm model from a representative Digital Subtraction Angiography (DSA) image and used CFD software to get the wall deformation and the velocity field. They have seen many differences between the two models by comparing the simulation results of the two models from their velocity vectors and shear stress distribution. They have also seen that the off-center distribution of velocity magnitude affects the distribution of wall shear stress. Ultimately they have concluded that the results of 2-D elastic numerical simulation are in good agreement with the clinical and the results of this study play an important role in the formation, growth, rupture and prognosis of an aneurysm on clinic application

.

**J. V. Ramana Reddy and D. Srikanth *et al.* [2016]** have performed an analysis to study the change in flow pattern and estimate the variation in flow resistance and wall shear stress in a narrow overlapping stenosed tapered arterywhen a catheter is inserted into it. They have performed the analysis assuming blood as steady incompressible micropolar fluid. They have solved the governing equation of fluid flow under the assumption of the mild stenosis using Mathematic and Matlab. Finally they have concluded that shear stress at the wall is increasing as micropolar parameter is decreasing and the trend isreversed in case of coupling number.

**K. Haldar *et at .* [1985]**  has carried out an analytical study on the effect of the shape of the stenosis on the resistance to blood flow through an artery with mild local narrowing. He has compared the resistance to flow with the impedance ratio and shape parameter. He has found that the impedance ratio increases as the length of the stenosis increases. Finally he has concluded that the resistance to flow decreases as the shape of the stenosis changes and the maximum resistance is attained in the case of symmetric stenosis.

**Kh. S. Mekheimer & M.A. El Kot *et al.* [2015]** have analysed a particle-fluid suspension model for the axi-symmetric flow of blood through curved coaxial tubes where the outer tube with mild overlapping stenosis while the inner tube is uniform rigid representing catheter. They have written the governing equations in a rectangular toroidal coordinates and formulated the problems in terms of a variant of curvature parameter to obtain explicit forms for the axial velocities of fluid and particulate phases, the stream function, the resistance impedance, pressure drop and the wall shear stress distribution. Also they have studied for various values of the physical parameters, such as the curvature parameter ε, the radius of catheter s, the volume fraction density of the particles C, the taper angle f and the maximum height of stenosis d\* . They have obtained the results that there is a significant deference between curvature and non-curvature annulus flows through catheterized stenosed arteries. Their study has provided a scope for estimating the influence of the problem parameters on different flow characteristics and to ascertain which of the parameters has the most dominating role.

**Kumar *et al*. [2014]** have carried out a numerical analysis to investigate with a non-Newtonian arterial blood flow model through multiple stenosis.They have shown the wall shear stress graphically for different values of hematocrit and viscosity of blood. They have compared the wall shear stress and pressure gradient for various cases with non-Newtonian data. They have concluded that the ratio of maximum height of stenosis and radius of normal artery and shear stress of the non-Newtonian fluid are strong parameter in fencing the blood flow. It has also seen that the wall shear stress decrease with the increase of blood viscosity.

**Li and Kleinstreuer *et al.* [2007]** have performed a numerical analysis to study the transient 3-D blood flow characteristics, pressure distribution and wall stresses employing computational fluid dynamics for an abdominal aortic aneurysm (AAA). They have noticed the impacts of the degree of asymmetry, neck angle and bifurcation angle on the hemodynamics and biomechanics. They have employed a coupled fluid-flow and solid–structure solver to obtain more realistic and accurate results for blood flow fields and wall stress distributions. The simulation results indicate that the assumption of symmetric AAA geometry may decrease AAA-wall stress considerably. A large neck angle results in strong wall curvatures near the proximal neck, produce aggravating blood flow patterns and elevated wall stresses. The iliac bifurcation angle do not affects blood flow patterns significantly but plays an important role in wall-stress concentrations. They have also seen that the wall stress of lateral asymmetric AAAs is higher than for the anterior-posterior asymmetric types.

**Ma and Turan *et al.* [2011]** have performed a numerical simulation in a 3D abdominal aortic aneurysm bifurcating model for non-Newtonian and pulsatile blood flow. They have found that the wall shear stress (WSS) is Changing significantly at both the proximal and distal ends of the aneurysm. They have seen that the WSS reaches to a peak value around the bifurcation point, whereas the WSS becomes zero in the bifurcation point, at the peak systole. They have noted that the geometry of the bifurcating region has significant influences on the upstream flow field. They have also observed a stronger secondary flow in the bifurcation zone.

**Musad Mohammed and Musad Saleh *et al.* [2011]** have developed a mathematical model to study the effect of paired stenosis on blood flow, where the blood flow is assumed to behave like a couple stress fluid, peripheral layer plasma (Newtonian fluid) and core layer of suspension of erythrcytes (Non-Newtonian fluid). They have solved the equation of wall shear stress of artery using Microsoft Mathematics 3.0. They have varied the ratio between height of stenosis and radius of artery from 0.1 to 0.9 and the ratio between viscosity coefficient of plasma layer and core layer varied from 1 to 2.5. They have seen that wall shear stress increases with the increase of one or both heights of stenosis. They have observed that the wall shear stress rang is 0.1 to 0.52 Pa, pressure gradient rang is 200 to 300 Pa/m and viscosity rang is 0.004 to 0.009 Pa-s.

**Neetu Srivastava [2014]**has analytically investigated about the MHD blood flow in a porous inclined stenotic artery under the influence of the inclined magnetic field. Considering blood as an electrically conducting Newtonian fluid he has described the physics of problem with the appropriate boundary condition by the usual MHD equations. Analytical expressions for the velocity profile, volumetric flow rate, wall shear stress, and pressure gradient have been derived. Graphically he has presented the blood fliw charecteristics for a specific set of value of the different parameters and transformed the equation to homogenious second order ordinary differtionial equation. Here he has shown that the flow of blood through stroids depends upon difference parameter like axial velocity, shear stress, magnetic field, angle of inclination of artery, volumetric flow of blood etc. Some of the obtained results show that the flow patterns in converging region (𝜉0), and nontapered region (𝜉=0) are effectively influenced by the presence of magnetic field and change in inclination of artery as well as magnetic field. There is also a significant effect of permeability on the wall shear stress as well as volumetric flow rate.

**Noreen Sher Akbar et al. [2015]** has analysed the blood flow through a tapered artery with a stenosis and also assumed the blood as tangent hyperbolic fluid modeld. They have analytically solved the nonlinear implicit system of partial differential equations with the help of perturbation method and also obtained the expressions for shear stress, velocity, flow rate, wall shear stress and longitudinal impedance. They have discussed the variations of power law index m, Weissenberg number, shape of stenosis n and stenosis size d in different type of tapered arteries and compared them with different parametes.

**Paramasivam *et al.* [2010]** have performed a numerical simulation in order to model clinically relevant hemodynamic conditions that are important for predicting the risk of rupture of AAAs. They have employed finite element method in the numerical simulation. The results from this method are validated by comparing the hemodynamic conditions reported in a typical AAA established experimentally. They have found a recirculating vortex in the aneurysm cavity which moves to the distal end as Reynolds number increases. They have also noted that the wall static pressure increases gradually in the cavity and reaches to a peak value which is always located downstream of it. They have also noted that the wall shear stress is negative in nature in the cavity and a peak wall shear stress at the exit.

**Qing *et al*. [2009]** have performed a numerical simulation on a three-dimensional statistical saccular model of a terminal aneurysm of the internal carotid artery. They have performed the simulation for both steady and pulsatile flow condition. They have studied the influence of aneurysm geometry on the local hemodynamics by changing the sac diameter and Aspect Ratio (AR) of the aneurysm. They have seen that the aspect ratio of the aneurysm gives more impact on the hemodynamics than the sac diameter. With increasing AR, the averaged WSS is significantly weakened and the wall pressure in the aneurysm is increases. They have concluded that the low WSS and high pressure of the aneurysm may play important roles in the fragile change of the aneurysm and the final rupture.

**Rabby *et al.* [2013]** have performed a numerical analysis on the physics of a pulsatile non-Newtonian flow confined within a two dimensional axisymmetric pipe with an idealized stenosis using the finite volume method. They have modified the governing Navier-Stokes equations using the cartesian curvilinear coordinates to handle the complex geometry such as arterial stenosis. The flow has characterised by the Reynolds number at 300,400 and 500 which are appropriate for the large arteries. The numerical results has presented in terms of the velocity, pressure distribution, wall shear stress as well as the streamlines indicating the recirculation zones at the post stenotic region. Finally they have concluded that at the centre of the stenosis the wall shear stress is very high.

**Ramachandra Rao *et al*. [1983]** has derived an equation governing the excess pressure for an axially tethered and stenosed elastic tube filled with viscous liquid, by introducing the elasticity of the tube through pressure-area relation. He has solved the equation numerically for large womersley parameter and the results are presented for different types of pressure-radius relations and geometries by prescribing an outgoing wave suffering attenuation at some axial point of the tube. For a locally constricted tube he has observed that the pressure oscillates more and generates sound on the down stream side of the cohstrics.

**R. N. Pralhad and D. H. Schultz *et al*. [2004]** have done a theoretical study on blood flow in a stenosed tube assuming blood flow to be represented by a couple stress fluid. Flow parameters such as velocity, resistance to flow and shear stress distribution have computed for different suspension concentrations (haematocrit). They have chosen the Reynolds number chosen for the computation purpose in ranging from 10 to 400 only. They have seen that the values of shear stress increases with the increase of stenosis height and decrease with the increase of couple stress parameters.

**Salsac *et al*. [2006]** have performed Particle image velocimetry (PIV) measurements in in-vitro aneurysm models with changing their geometric parameters systematically. They have seen that even at the very early stages of the disease, i.e. increase in the diameter ≤50 %, the flow separates from the wall and a large vortex ring is created. They have also noticed that the mean WSS becomes negative along most of the aneurysmal wall inside the AAA and the magnitude of the WSS can be as low as 26% of the value in a healthy abdominal aorta.

**Santabrata Chakravarty *et al.* [1987]** has performed an analytical study to examine the effect of stenosis on vascular deformability and the flow of a non- Newtonian viscous incompressible fluid (blood) in an artery through the use of a suitable mathematical model. He has assumed the blood flow to becharacterized by the power law model. He has compared the blood flow rate with the power law exponent, vessel wall displacement with thickness of the stenosis at a particular location and resistance to flow with power law exponent at the critical point of the stenosis. Finally he has concluded that the deformability of the vascular wall has a greater effect on flow disorder and consequently the resistance to flow through the artery in the presence of stenosis.

**Sarifuddin *et al.* [2009]** have investigated with mathematical model representing the mass transfer to blood streaming through the arteries under stenoic condition. They have treated blood flow through the artery to be Newtonian and the arterial wall to be rigid having differently shaped stenoses. They have solved the governing equations of motion with appropriate boundary conditions numerically by MAC method and have checked numerical stability with desired degree of accuracy. They have performed the analysis with Reynolds number 300, Scanton number 3 and arterial length of 38.6mm. They havr noticed that the pressure drop appears significantly higher in the case of cosine shape of the stenosis and moderately higher for smooth stenosis with respect to the irregular one.

**Satyasaran Changdar & Soumen De *et al.* [2015]** have investigated the non-linear blood ﬂow under the inﬂuence of periodic body acceleration through a generalized multiple stenosed artery with the help of numerical simulation.The arterial segment is simulated by a cylindrical tube ﬁlled with a viscous incompressible Newtonian ﬂuid described by the Navier–Stokes equation. Numerically they have solved the non-linear equation using ﬁnite difference with the proper boundary conditions and pressure gradient that arise from the heart anfd also discussed the effect of Reynolds number.

**S. Chakravarty and A. Ghosh Chowdhury *et al.* [1988]** have done an analytical study on behaviour of blood flow in an artery having a stenosis assuming blood as a viscous incompressible Newtonian fluid. They have analysed the variation of the phase velocity, as well as the velocity of wave propagation and the flow rate. They have found that the stenosis creates disturbances in the resistance to flow of blood even outside the stenotic region. Finally they have concluded that the resistance to flow decreases as the shape parameter of the stenosis increases in case of symmetric stenosis.

**Sheard *et al.*  [2009]** has performed a numerical simulation in order to determine the flow dynamics and wall shear-stress for a pulsatile flow through a human abdominal aortic aneurysm model. He has employed a high-order spectral-element algorithm to accurately determine velocity, vorticity fields and wall shear stresses. He has also compared the numerical result with particle image velocimetry experiments. He has found that the wall shear stress is minimum at the widest point of the aneurysm bulge and maximum at the distal (downstream) region of the bulge. Finally he has conclude that the peak instantaneous wall shear stress for aneurysm dimensions (LR = 2.9, DR = 1.9), is 2.4 times greater than the peak wall shear stress in a healthy vessel.

**Sheard *et al.* [2009]** has performed a numerical simulation to study the flow behavior of a pulsatile flow within a three-dimensional fusiform abdominal aortic aneurysm model using a high-order spectral-element/Fourier method. He has investigated the simulation for a Reynolds number of 330, a Womersley number of 10.7, and aneurysm dimensions (LR = 2.9-5.2, DR = 1.3-5.2). He has computed the variation in wall shear stress as a function of both time and aneurysm dimension. He has found that the spatio-temporal fluctuation of wall shear stress throughout the pulse cycle which is increasing in nature.

**Shukla *et at .*[1980]** have investigated the effects of stenosis on resistance to flow and wall shear stress in an artery by considering the blood as power law and casson model fluids. They varied non dimensional resistance to flow with the ratio of maximum height of the stenosis to the radius of the normal artery for different artery length and yield stress. They have concluded that the resistance to flow and wall shear increase as the size of the stenosis increases for a given non- Newtonian model of the blood.

**Shukla *et al.* [1980]** have performed an analytical study about the effect of peripheral layer viscosity on physiological characteristics of blood flow through the artery with mild stenosis. They have expressed for the resistance to flow and plotted for different values of viscosities of the peripherallayers and the ratio of the length of the stenosis to the length of the artery. Finally they have concluded that the resistance to flow and the wall shear decrease as the peripheral layer viscosity decreases.

**Shupti *et al*. [2013]** have performed numerical simulation to study the flow behavior of a pulsatile Newtonian fluid confined within a two-dimensional (2D) channel with an asymmetric shaped aneurysm using the finite volume method. The investigation have been carried out to characterize the blood flow for Reynolds number 100, 300 and 500 which is suitable for human arteries. They have investigated hemodynamic factors that generally contribute to the rupture of AAA i.e., velocity, pressure distribution, wall shear stress as well as the streamlines indicating the recirculating zones inside the dilated region. They have found a lower centerline velocity inside the aneurysm but comparatively higher wall pressure and wall shear stress at the distal neck of the aneurysm. Finally, they have concluded that a higher Reynolds number has greater effects on the flow characteristics which affects the growth of aneurysm.

**S. P. Nanda and B. Basu Mallik *et al.* [2012]** have carried out atheoretical analysis to study blood flow through arteries under stenotic conditionfor a non-Newtonian two phase fluid model. They have solved the coupled differential equations governing the flow of fluid (plasma) and the particle phases have solved by a combined use of analytical and numerical techniques with appropriate boundary condition. They have plotted variation of impedance with non dimensional stenosis height for different C. They have observed theincreasing behaviour of the flow resistance when the stenosis height as well asthe hematocrit increases. They have concluded that the shear stress at stenosis throat and at critical height increases for increasing value of stenosis height in the permissible range of hematocrit.

**Stamatopoulos *et al*. [2010]** have carried out a research on numerical and experimental investigation to study the flow field for steady and unsteady flow within an axisymmetric tube dilatation model. They have employed a 2D PIV system for experimental and the commercial software FLUENT for numerical analysis. They have varied the Re in the range of 100-700 for the steady case. They have found that the recirculation zone length increases and the flow reattachment line being displaced towards the exit of the model with the increase of Re. They have also noted that the wall pressure peaks and wall shear takes a local maximum at the model exit. For unsteady case, they have taken the flow rate is sinusoidal, the Womersley number is 3.3 and peak Re = 272. They have seen that during early acceleration, a vortex ring is formed at the proximal part of the cavity and two stagnation points appear on the longitudinal axis of the model approaching each other as time progresses, eventually disappearing when the majority of the fluid particles changes direction.

**Tang *et al*.[2003]** have performed a nonlinear three-dimensional thick-wall model with fluid-structure interactions is introduced to simulate blood flow in carotid arteries with an asymmetric stenosis to quantify the effects of stenosis severity, eccentricity, and pressure conditions on blood flow and artery compression (compressive stress in the wall). They have used a thick-wall stenosis model made of polyvinyl alcohol hydro gel whose mechanical properties are close to that of carotid arteries to measured the mechanical properties of the tube wall. They have also used A hyper elastic Mooney–Rivlin model to implement the experimentally measured nonlinear elastic properties of the tube wall and the Navier– Stokes equations in curvilinear form for the fluid model. After getting the result they have indicated that severe stenosis causes critical flow conditions, high tensile stress, and considerable compressive stress in the stenosis plaque which may be related to artery compression and plaque cap rupture. In their results, stenosis asymmetry leads to higher artery compression, higher shear stress and a larger flow separation region.

**Valencia and Solis *et al.* [2006]** have performed a numerical investigation on the flow dynamics and arterial wall interaction in a representative model of a terminal aneurysm of the basilar artery. They have compared the effect of wall shear stress, pressure, effective stress and wall deformation with those of a healthy basilar artery by assuming the arterial wall to be elastic or hyperelastic, isotropic, incompressible and homogeneous. They have solved the fully coupled fluid and structure models were with the finite elements package ADINA by assuming the flow as laminar, Newtonian, and incompressible. The flow dynamics in the aneurysm model shows an unsteady vortex structure. They have found large spatial and temporal variations of pressure and shear stress on the wall of the aneurysmal artery.

**Vipin Kumar Verma and Praveen Saraswat *et al.*[2013]** have done a theoretical analysis to analyze the characteristics of blood flow (steady) through an artery with axi symmetric multiple stenosis in series considering blood as a Newtonian fluid. They haveplotted pressure drop across first stenosis length and second stenosis length. They have found out shear stress on the stenosis surface and seen that maximum value of shear stress in the middle of first and second stenosis and minimum at the ends of stenosis. Finally they have concluded that only small values of non dimensional stenosis height the effect of the stenosis can be predicted from measurement of the individual stenosis.

**Xin-yu *et al*. [2001]** have performed a numerical analysis of Newtonian and non-Newtonian flow in an axi-symmetric tube with a local constriction simulating a stenosed artery under steady and pulsatile flow conditions have been carried out. They have discussed the concentration fields of LDL ( low- density lipoprotein ) and Albumin based on the results. According to their results, the macromolecule transport influences of wall shear stress, non-Newtonian fluid character and the scale of the molecule. They have compared the result between Newtonian fluid flow and non- Newtonian fluid flow, steady flow and pulsatile flow. After the investigation, they have provided much valuable information about the correlation between the flow properties, the macromolecule transport and the development of atherosclerosis.

**Zhang *et al*. [2007]** have carried out a numerical and experimental investigation for a pulsatile flow fields in rigid abdominal aortic aneurysm (AAA) models. They have found that there are one or more vortexes are formed in the AAA bulge and a fairly high wall shear stress exists at the distal end. They have also developed a porous medium computational model. They have shown that the wall shear stress can be minimized with that model and there is less danger of rupture. They have also verified numerically the therapeutic effect of the stent-graft with the new model.

**Objective**

After comprehensive survey of literature, it has been observed that the flow characteristic of biological fluids through stenosis arteries have some significant effects on the initiation and progression of the disease. So, before getting to treatment procedure, it is very crucial to know the causes and characteristic of the disease. In this situation, the application of engineering methodologies can be quite effective. So, it is very important to know the flow characteristic of blood passing through an stenosis artery from a numerical point of view. But we have not found any literature which has shown both the effects of dilation ratio and aspect ratio on the flow characteristic of blood passing through stenosis arteries. The objectives of this work are follows:

* To numerically predict the flow characteristics such as streamline contour, recirculating bubbles, wall static pressure, wall shear stress and maximum wall shear stress gradient of the blood passing through a an stenosis artery with the variation of Reynolds number, dilation ratio (DR) and aspect ratio (AR).
* To study the effect of flow characteristics on the initiation and progression of the disease stenosis.

**Mathematical** Formulation

**4.1 Computational domain:**

A rigid, symmetric AAA model is used in this study, as computational domain. The schematic diagram of the computational domain is illustrated in fig. 4.1. Although the models are not physiologically correct in geometry, they still possess all the important physical processes that occur in an AAA. We have considered inner diameter of artery, d=20 mm all the cases. Here, D is the maximum bulge diameter and L is the length of the bulge.

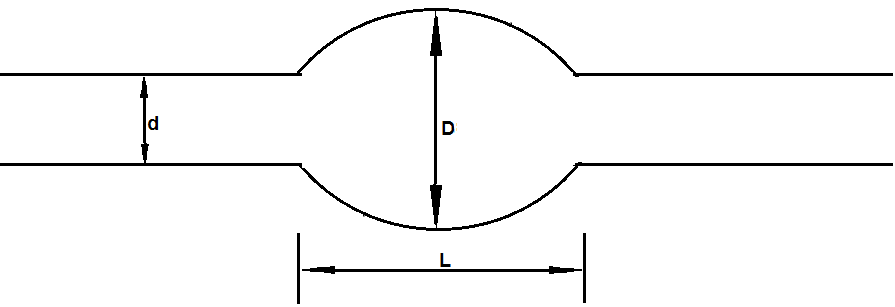


Fig 4.1: Schematic diagram of computational domain.

In the current study, the geometrical shapes are characterized by two geometric parameters: the dilation ratio (DR=D/d) and the aspect ratio (AR=L/d). To understand the basic effects of aneurysm shapes on the hemodynamic stresses, the following aneurysm geometries are considered:

* The dilation ratio (DR) of the model has been varied as 1.5, 2, and 2.5, keeping the aspect ratio (AR) constant as 2.5.
* The aspect ratio (AR) of the model has been varied as 1.5, 2, 2.5, 3, 3.5, and 4, keeping dilation ratio (DR) constant as 2.5.

**4.2 Governing Equations:**

**Assumptions:**

The flow under consideration has been assumed to be steady, two-dimensional and laminar. Here, the fluid (blood) has been considered to be incompressible. Although blood has actually non-Newtonian behavior, in the simulation it is considered as Newtonian fluid [Scotti and Finol, 2007]. We have taken density (ρ**)** = 1060 kg/m3 and dynamic viscosity = 0.0035 kg/ms.

Under this assumption the continuity and Navier-Stokes equations can be written as follows:

**Continuity equation:**

(4.1)

**Momentum equation:**

r-direction momentum equation:

(4.2)

z-direction momentum equation:

(4.3)

Where, is velocity in radial direction, is the velocity in axial direction, p is pressure, is density, is the coefficient of dynamic viscosity.

**4.3 Boundary conditions**

Three different types of boundary conditions have been applied to the present problem. They are as follows,

(i) At the walls: No slip condition, *i.e.* , .

(ii) At the inlet: Axial velocity has been specified and the transverse velocity has been set to zero, *i.e*. , .

(iii) At the exit: Constant pressure boundary has been adopted.

**4.4 Numerical Procedure**

The geometry and mesh of the computational domain have been made in commercial software GAMBIT 2.3.16 which is the preprocessor for FLUENT 6.3.26. The numerical grid is unstructured in the bulge area and structured in the straight tubes. The Nevier-Stokes equation and the continuity equation are solved using the commercial CFD software FLUENT 6.3.26 that employs the control volume technique on a uniform staggered grid following SIMPLE algorithm. The convective terms are discretized by upwind scheme. The convergence of the iterative scheme is achieved when the normalized residuals of mass and momentum equations summed over the entire calculation domain fall below 10-5.

**4.5 Grid Independence**

In this investigation, wall shear stress used to illustrate grid independence. Fig. 3.2 illustrate the comparison of wall shear stress at Re = 400, dilation ratio (DR) 2, and aspect ratio (AR) 2.5.



Fig. 4.2: Comparison of WSS for different quadrilateral elements at Re=400 for DR=2.

It is very important to show grid independence when doing computational studies. To illustrate more and to show the grid independence, three models with different quadrilateral elements have been simulated. The quadrilateral elements are selected as 30940, 123324, and 192819. It can be seen in fig. 3.2 that the wall shear stress values for all the three cases are almost similar and follow a similar pattern. The model with 192819 quadrilateral elements exhibited slightly higher shear stress at the distal end of the aneurysm, but it consumed a considerable amount of time. For this simulation, the model with 123324 quadrilateral elements has been simulated. The model with 192819 quadrilateral elements was not selected to avoid time consumption and memory storage.

**REFERENCES**

Srivastava N., 2014, Analysis of Flow Characteristics of the Blood Flowing through an Inclined Tapered Porous Artery with Mild Stenosis under the Influence of an Inclined Magnetic Field, Journal of Biophysics, 2014, 1-9.

Changdar S., De S., 2015, Analysis of Non-linear Pulsatile Blood Flow in Artery through a Generalised Multiple Stenosis, Arabian Journal of Mathematics, 5, 51-61.

Tang D., Yang C., Kobayashi S., Zheng J., Vito R. P., 2003, Effect of Stenosis Asymmetry on Blood Flow and Artery Compression: A Three-dimensional Fluid-Structure Interaction Model, Annals of Biomedical Engineering, 31, 1182-1193.

Xin-yu L., Gong-bi W., Ding L., 2001, Computer Simulaton of Non-Newtonian Flow and Mass Transport through Coronary Arterial Stenosis, The National Natural Science Foundation of China, 22, 409-424.

Tashtoush B., Magableh A., 2007, Magnetic Field Effect on Heat Transfer and Fluid Flow Charecteristics of Blood Flow in Multi-stenosis Arteries, Heat Mass Transfer, 44, 297-304.

Riahi D. N., 2016, Modeling Unsteady Ttwo-phase Blood Flow in Catheterized Elastic Artery with Stenosis, Engineering Science and Technology, an International Journal, 19, 1233-1243.

Rao A. R., 1983, Unsteady Flow with Attenuation in a Fluid Filled Elastic Tube with a Stenosis, ACTA Mechanica, 49, 201-208.

Gataulin Y. A., Zaitsav D. K., Smirnov E. M., Fedorova E. A., Yukhnev A. D., 2015, Weakly Swirling Flow in a Model of Blood Vessel with Stenosis: Numerical and Experimental Study, St. Petersburg Polytechnical University: Physics and Methamatics, 1, 364-371.

Khan M. Y. A., 2011, Eﬀect of Paired Stenosis on Blood Flow through Small Artery, Journal of Mathematics Research, 3, 224-229.

Srivastava N., 2014, Analysis of Flow Characteristics of the Blood Flowing through an Inclined Tapered Porous Artery with Mild Stenosis under the Influence of an Inclined Magnetic Field, Journal of Biophysics, 2014, 1-9.

Reddy J. V. R., Srikanth. D, 2015, The Polar Fluid Model for Blood Flow through a Tapered Artery with Overlapping Stenosis: Effects of Catheter and Velocity Slip, Applied Bionics and Biomechanics, 2015, 1-12.

Chakravarty S., Chowdhury A. G., 1988, Response of Blood Flow through an Artery under Stenotic Conditions, Rheologica Acta, 27, 418-427.

Haldar K., 1985, Effects of the Shape of Steneosis on the Resistance to Blood Flow Through an Artery, Bulletin of Mathematical Biology, 47, 545-550.

Shukla J. B., Parihar R. S., Rao B. R. P., 1980, Effects of Stenosis on Non-Newtonian Flow of the Blood in an Artery, Bulletin of Mathematical Biology, 42, 283-294.

Sarifuddin., Chakravarty S., Mandal L. K., 2009, Mass Transfer to Blood Flowing Through Aarterial Stenosis, Zeitschrift fur angewandte Mathematik und Physik, 60, 299-323.

Ellahi r., Rahman S. U., Nedeem S., Akbar N. S., 2013, Blood Flow of Nanoﬂuid through an Artery with Composite Stenosis and Permeable Walls, Appl Nanosci (2014) 4, 919–926.

Varma V. K., Saraswat P., 2013, Effect of a Multiple Stenosis on Blood Flow through a Tube, World Academy of Science, Engineering and Technology International Journal of Medical, Health, Biomedical, Bioengineering and Pharmaceutical Engineering, 7, 553-556.

Nanda S. P., Mallik B. B., 2011, A Non-Newtonian Two-Phase Fluid Model for Blood Flow Through Arteries Under Stenotic Condition, International Journal of Pharmacy and Biological Sciences, 2, 237-247.

Achab L., Manfoud M., Benhadid S., 2016, Numerical Study of the Non-Newtonian Blood Flow in a Ssneosed Artery Using Two Rheological Model, THERMAL SCIENCE, 2016, 20, 449-460.

Gataulin Y. A., Zaitsav D. K., Samirnov E. M., Fadorova E. A., Yukhnev A .D., 2015, Weakiy Fwirling Flow in a Model Blood Vessel with Stenosis Numerical and Experimental Study, St Petersburg Polytechnical Univrersity Journal: Physics and Mathematics, 1, 364-371.

Akbar N .S., 2015, Non Newtonian Model Study for Blood Flow through a Tapered Artery with Stenosis, Alexandria Engineering Journal, 55,321-329.

Chakravarty S., 1987, Effects of Stenosis on the Flow Behavior of Blood in Artery, Pergamon Journal Ltd, 25, 1003-1016.

Mekheeimer Kh. S. Kot M. A. El., 2015, Suspension Model for Blood Flow through Catheterized Curved Artery with Time-variant Overlapping Stenosis, Engineering Science and Technology, an International Journal, 18, 452-462.